for water when the energy input is 1 hp. per 100 gal. Assuming ϵ and the energy input per unit volume are proportional, then we may rewrite Equation (4) and find η (in microns) from

$$\eta = 25(\epsilon')^{-1/4}\nu^{3/4} \qquad (11)$$

where ϵ' is in hp. per 100 gal. and ν is in centistokes. Values of η are shown in Table 2.

Figure 1 shows a test of Equation (10), for $a = 1/2$ and $b = 2/3$. The large and small particle data are distinguished by using open or filled symbols, respectively. Considering the wide range of variables, the correlation is seen to be fairly good. A similar test of Equation (2)/(6) yielded separate lines for each set of data, with the separation being as much as an order of magnitude in the abscissa. It would appear that a model based on Equation (2)/(6) may be rejected.

A slope of unity for the large particle data would be in agreement with Equation (1)/(6). The slope of the upper portion of the curve is found to be about 1.2. A slope of unity for the small particle data would confirm Equation (2)/(8). The slope of the lower portion of the curve is about 0.75. Equations (1)/(7) and (2)/(7) predict slopes of 0.75 ond 0.67, respectively, for the lower portion of the data.

The values of η given in Table 2 are, at best, order of magnitude estimates. Thus, we are in no position to argue about where one theory is applicable compared with another, with respect to particle size. Furthermore, the small amount of data, and the scatter in these data, make it difficult to argue for a slope of 0.75 compared to a slope of 0.67. Hence, we refrain from any attempt to argue further among the individual correlations. We do observe, however, that the Frössling equation in conjunction with the Kolmogoroff theory predicts a correlation in reasonable agreement with the data at hand.

It is appropriate to remark that the application of the Kolmogoroff theory to mass transfer from particles is not a new idea. Levich (*9*) presents results that can be shown equivalent to Equations (1)/(8) and (3)/(6), but examines no data.

Calderbank (*2*) applies the theory, but fixes the exponent of the Reynolds number to agree with the "observation" that k is independent of D_P. The data of Barker and Treybal (*1*) are cited in support of this observation. Their particle sizes in a given run varied by a factor of two or three and were

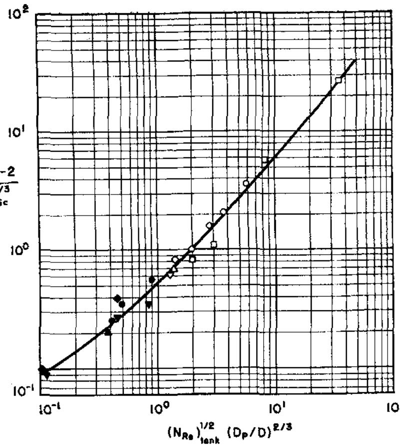

Fig. 1. Harriott's data (his Figure 8) replotted according to Equation (10), with $a = 1/2$ and $b = 2/3$. Ion exchange resin in: ○ water; △ 6.7 cp methocel; ▽ 20 cp methocel; ◇ 18 cp glycerine. Benzoic acid in water □. Filled symbols represent data for $D_p < \eta$; open symbols for $D_p > \eta$.

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First-Order Reaction and Anisotropic Diffusion in Flowing Media

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Studies on the mass transfer in fluids moving through porous materials suggest that it may be useful to introduce anisotropic effective diffusivities, represented by a tensor quantity (*1*). Thus the molar flux density of a component A, referred to stationary coordinates (noncoincident with the principal axes of diffusion), should be

$$N_{Ai} = -D_{ij}\frac{\partial c_A}{\partial x_j} + c_A u_i \qquad (1)$$

where $i, j \ldots .$ are dummy suffixes indicating the components of vectors and tensors along the axes, and Einstein's summation convention is adopted.

If the component A reacts with a first-order kinetics, the continuity equation, introducing expression (1), becomes

$$\frac{\partial c_A}{\partial t} = -u_i\frac{\partial c_A}{\partial x_i} + \frac{\partial}{\partial x_i}D_{ij}\frac{\partial c_A}{\partial x_j} - \left(k + \frac{\partial u_i}{\partial x_i}\right)c_A \qquad (2)$$

where, to maintain the generality, fluid density is not assumed constant.

In previous papers (*2* to *4*) Danckwerts' relation (*5*), giving the concentration profiles for unsteady diffusion in systems with first-order reaction in terms of those for nonreacting systems of the same geometry and with identical boundary conditions, is extended to flowing media with constant density and diffusivity.

Following the development given in the previous papers (*2* to *5*), it is easy to verify that if it is

$$\frac{\partial \overline{c}_A}{\partial t} = L\,\overline{c}_A \qquad (3)$$

$$\overline{c}_A\,(0, \mathrm{x}) = 0 \qquad (4)$$

where L is an operator depending only upon position and containing only spatial derivatives, Dankwerts' relation

$$c_A = k\int_0^t \overline{c}_A\,(\lambda, \mathrm{x})\,e^{-k\lambda}\,d\lambda + \overline{c}_A\,e^{-kt} \qquad (5)$$

gives an integral of the equation

$$\frac{\partial c_A}{\partial t} = (L - k)\,c_A \qquad (6)$$

where k is a constant. Obviously it is

$$c_A\,(0, \mathrm{x}) = 0 \qquad (7)$$

It is also easy to verify that if $\overline{c}_A$ satisfies boundary conditions of the following types

$$\overline{c}_A\,(t, \mathrm{x}) = c_o\,(\mathrm{x}) \qquad (8)$$

$$\lambda_i D_{ij}\frac{\partial \overline{c}_A}{\partial x_j} = 0 \qquad (9)$$

$$-D_{ij}\frac{\partial \overline{c}_A}{\partial x_j} + \overline{c}_A u_i = f_i\,(\mathrm{x}) \qquad (10)$$

the same conditions are satisfied by c_A.

Thus the solution of Equation (2), using (7), (8), and (9), may be ob-

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INFORMATION RETRIEVAL

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Studies of boiling heat transfer with electrical fields: Part I. Effect of applied a.c. voltage on boiling heat transfer to water in forced circulation. Part II: Mechanistic interpretations of voltage effects on boiling heat transfer, Markels, Michael, Jr., and Robert L. Durfee, **A.I.Ch.E. Journal, 11,** No. 4, p. 716 (July, 1965).

Key Words: A. Boiling-8, 7, Heat Transfer-8, 7, Water-1, Voltage-6, Alternating Current-6, Film-9, Forced Convection-10, Resistivity-6, Dielectrophoresis-8. B. Heat Transfer-7, Water-1, Voltage-6, Surface Wetting-7, 6, Boiling-8, Isopropanol-1, Alcohol-1, Mixing-6, Bubbles-6.

Abstract: Alternating current voltage was found to increase boiling heat transfer rates and prevent film boiling for deionized water under forced convection at essentially ambient pressure.

An analysis of surface wetting during the application of voltage to the boiling process for isopropanol and water was made using current and voltage measurements from previous work. In the normal film boiling region, surface wetting initially contributes strongly to the increased heat transfer and, at higher voltages, other factors such as local mixing and bubble characteristics appear to predominate.

Model simulation of stirred tank reactors, Manning, F. S., David Wolf, and D. L. Keairns, **A.I.Ch.E. Journal, 11,** No. 4, p. 723 (July, 1965).

Key Words: Yields-8, 9, 7, Stirred Tank Reactors-9, Reactors-9, Prediction-8, Model Simulation-10, Residence Time-6, Size-6, Impeller-9, Velocity-6, Temperature-7, Outlet-9, Mixing-8, Comparison-8, Experimental-0, Theoretical-0, Water-5, Rate-6, Feed-9, Concentration-6.

Abstract: Stirred tank reactor yields are successfully described by the following model. The impeller is considered to act as a local, micromixer that perfectly mixes the recirculating stream down to the molecular level. All other portions of the vessel act as a large volume macromixer, throughout which the impeller discharge stream remains completely segregated. Changes in overall conversion due to variations in mean residence time, impeller size, and rev./min., as predicted by this micro- and macromixer model, agree with Worrell's data for a relatively slow, second-order, irreversible reaction.

Size distribution dynamics in a salting out crystallizer, Murray, D. C., and M. A. Larson, **A.I.Ch.E. Journal, 11,** No. 4, p. 728 (July, 1965).

Key Words: A. Nucleation-6, Growth Kinetics-6, Operating Conditions-6, Size Distribution-7, 8, Crystals-9, Salting Out Crystallizer-9, 8, Crystallizer-9, 8, Ammonium Alum-2, Ethanol-5, Alcohol-5, Water-5, Experimental-0, Steady State-0, Unsteady State-0.

Abstract: A laboratory continuous mixed suspension salting out crystallizer was designed and constructed to test the dynamic crystallization model proposed by Randolph and Larson. The model relates the crystal size distribution to nucleation and growth kinetics and operating conditions. The ammonium alum-ethanol-water system was selected because of its ease of operation and control. The crystallizer was operated under steady and unsteady state conditions.

The numerical solution of boundary-layer problems, Fussell, D. D., and J. D. Hellums, **A.I.Ch.E. Journal, 11,** No. 4, p. 733 (July, 1965).

Key Words: A. Boundary-Layer Theory-8, Solution-8, Boundary Layer-9, Computers-10, Goertler-Transformed Equations-10, Finite Difference Methods-10, 7, Nonlinear-0, Numerical Analysis-10, Applied Mathematics-10, Heat Transfer-9, Fluid Mechanics-9, Transport Phenomena-9, Laminar Flow-9, Mass Transfer-9, Viscous Dissipation-6, Viscous Heating-6, Physical Properties-6.

Abstract: A new method for numerical solution of boundary-layer problems is presented. The new method is shown to have important advantages over the several previously proposed methods in range of applicability as well as in speed or accuracy. The method uses the Goertler-transformed equations and a nonlinear finite difference procedure. Typical complete solutions are given with comparisons of the several methods. The solutions include both similar and more general flows up to separation, as well as coupling between the momentum and energy balance equations.

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tained is terms of that for the nonreacting system applying Danckwerts' relation (*5*), when u_i and D_{ij} depend only upon the position, k is constant, and the initial concentration is null.

When the boundary conditions are given by Equations (8) and (9) it is possible (*2, 4*) to obtain the solution for non-null initial conditions, adding to the Danckwerts' relation the term

$$c_{A1}\, e^{-kt} \qquad (11)$$

where c_{A1} is the solution of Equation (2), with $k = 0$ satisfying the considered initial conditions and the conditions of null concentration, or its spatial derivatives, on the boundary surface.

It is clear that the interest of relations giving the concentration profiles for reacting system in terms of those for nonreacting ones is not confined to the cases where the nonreacting system equations are integrable. Indeed, in many cases it may be interesting to calculate concentration profiles for reacting systems from experimentally determined concentration profiles for nonreacting systems.

Lastly, Equation (2), or its particular cases, may be found in the study of mass transfer in anisotropic turbulence fields with a first-order reaction.

NOTATION

c_A = molar concentration of reagent A for reacting system
$\bar{c}_A$ = molar concentration of reagent A for nonreacting system
c_{A1} = molar concentration of reagent A for nonreacting system with non-null initial concentration and null concentration, or its spatial derivatives, on the boundary surface
D_{ij} = diffusion coefficients
k = specific reaction rate
N_{Ai} = component along axis x_i of the molar flux density
t = time
u_i = component along axis x_i of the velocity
x_i = coordinate axis
λ_i = direction cosine of the normal to boundary surface

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